A Simple Statistical Model for QGP Phenomenology .

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1 Abstract:

We propose a simple statistical model for the density of states for quarks and gluons in a QGP droplet, making the Thomas-Fermi model of the atom and the Bethe-model for the nucleons as templates for constructing the density of states for the quarks and gluons with due modifications for the 'hot' relativistic QGP state as against the 'cold' non-relativistic atom and nucleons, which were the subject of the earlier 'forebears' of the present proposal.We introduce 'flow-parameters' $\gamma_{q,g}$ for the quarks and the gluons to take care of the hydrodynamical (plasma) flows in the QGP system as was done earlier by Peshier in his thermal potential for the QGP. By varying γ_g about the 'Peshier-Value' of $\gamma_q = 1/6$, we find that the model allows a window in the parametric space in the range $8\gamma_q \leq \gamma_g \leq 12\gamma_q$, with $\gamma_q = 1/6$ (Peshier-Value), when stable QGP droplets of radii $\sim 6\ fm$ appear at transition temperatures $100\ MeV \leq T \leq 250\ MeV$. The smooth cut at the phase boundary of the Free energy vs. droplet radius suggests a First - Order phase transition .On the whole the model offers a robust tool for studying QGP phenomenology as and when data from various ongoing experiments are available .

The conjecture that the constituent quarks and gluons in hadrons can remain in a deconfined phase is one of the exciting ideas in high energy physics with significance in ultra relativistic heavy ion collisions, interior of massive neutron stars, and possibly in the early phase of the 'Big-Bang' model of universe. Therefore, the transition between the two phases, viz., the quark-gluon plasma (QGP) phase and the hadronic phase is important from the theoretical point of view.

At present a rigorous QCD treatment of the problem is almost impossible, given the complexity of the physical system involved. However, lattice QCD studies have given indication of the transition being a first order one in the Gibbs sense, though not yet definitively for physical QCD.

In the meanwhile, a number of papers using some phenomenological models have appeared over the past decade investigating the phase transition between hadronic and QGP phases. Of interest to us here are the work of Mardor and Svetitsky [1] and more recently of Neerguard and Madsen [2] who used the MIT bag model for the hadrons and also invoked the idea of zero chemical potential case in the computation of free energy. A direct numerical calculation of Free energy of a QGP droplet in a bulk hadron (pionic) medium of radius R, led Mardor and Svetitsky [1] to conclude that the free energy of the system indeed behaves as in a first order phase transition, i.e. the free energy F(R) as a function of R has a minimum at R=0 when $T < T_0$ (the transition temperature) and the true minimum ($F \to -\infty$) and $R \to \infty$, with a small energy barrier in between at $R < 5$ fm. Neerguard and Madsen [2] use a more elaborate calculation by introducing a 'Concentric-sphere' model to evaluate the density of states of quarks and gluons and also make a comparative analysis using the 'multiple reflection expansion' approach of Balian and Block [2] for the same. This analysis also supports the overall conclusion of Mardor and Svetitsky [1] regarding the nature of transition.

Our intention in this paper is to reevaluate the free energy of a QGP droplet in a bulk hadronic (pionic) medium, again in the limit of vanishing chemical potential, but using a different semi-phenomenological model for the system. The M.I.T. bag model is simplicity itself; it puts all quarks and gluons as free particles inside a bag and makes the impermeable bag as the agent of confinement

by ascribing a set of boundary conditions for quarks and gluons. It is fine to use the M.I.T. bag model to describe the hadrons as bags of quarks, antiquarks and gluons, but to extend the idea to represent the phase boundary between the QGP droplet and the bulk hadronic medium makes one a bit uneasy. And, this is precisely the assumption made by the earlier authors who have used the M.I.T. bag model for the system of QGP droplet in a bulk hadronic medium. It is to remedy this rather unnatural assumption, i.e. the confining bag of the hadrons has the same property as the interface separating the two phases, we propose an alternative model to represent the same physical situation.

Another drawback of the M.I.T. bag model is its disagreement with "numerical experiments" using lattice guage pure SU(3) simulations [3]. As pointed out by Peshier et. al [4], the simulation "data" is satisfied only by a bag pressure $p = ae - \frac{4B}{3}$, with a=0.297 (not 1/3 as for M.I.T. bag model !), where e is the energy density and $B^{1/4} = 205MeV$ for the bag constant. This mismatch with lattice simulations had led earlier authors to abondon the M.I.T. bag model in the context of Q.G.P. by introducing "thermal parton masses" [4,5].

Central to the computation of the free energies of the de-confined constituents of the Q.G.P. and its surrounding hadronic medium, is the computation of their respective density of states for which several models like the phase shift model, the multiple reflection expansion of Balian and Bloch [2] etc., have been employed as mentioned earlier, with several "hand-waving" arguments to arrive at tractable results. We present here a simple model for the quarks density of states, which can be suitably modified for the case of gluons. Since the model depends crucially on the nature of the effective semi-phenomenological, Q.C.D. oriented [in so far as the form of the Q.C.D. running coupling constant is concerned] potential between quarks that we extract from the large momentum approximation to the "thermal mass" introduced by Peshier et. al [4], and adopt their phenomenological parametrisation in our scheme.

It will be in order to mention that our earlier attempt at using the model with a "Cornell-potential" [7,8] did not lead to very good results and was also open to the criticism that the "Cornell-potential" was not compatible with the dynamical (thermal) nature of the quarks and gluons in Q.G.P. It is with all these

foregoing considerations that we use an effective thermal potential extracted from the "thermal mass" form (or the "thermal-Hamiltonian") of Peshier et. al [4], whose statistical mechanical moorings were shown to be on sound footing by Gorenstein and Yang [4]. In this model the effect of the intercation is taken care of by the density of states, while quarks and gluons can be treated as non-interacting particles for all practical purposes. Our assumpion is borne out by the numerical evaluation of the Free-energies.

2 A modified 'Thomas-Fermi' model for the QGP droplet

In a very elegant and successful statistical model of atoms of large atomic numbers Thomas and Fermi [6] demonstrated the way to compute electronic density of states to very high order of accuracy. The Thomas Fermi model of atom assumes the electrons to be Fermi-Dirac gas confined within a localized region by the confining electrostatic potential V(r) of the central nucleus. The potential is assumed to be very slowly varying in the region with the average thermal energy T (setting the Boltzmann constant to unity) is small compared to V(r) within the region and comparable to it near boundary.

It is now straight forward [6] to compare the electronic density of states, assuming all states to be filled in a volume ν

$$N_e = p_{max}^3 \, \nu/3\pi^2 \tag{1}$$

The maximum kinetic energy of the electron at any point in phase space should not exceed the electrostatic potential (confining) at that point and therefore $p_{max}^2/2m = -V(k)$, when k is the phase point under consideration and $V(k)$ is the momentum transform of the coordinate potential $V(r)$. Therefore, the total density of states in phase space is given by

$$\int \rho_e(k)dk = [-2mV(k)]^{3/2}\nu/3\pi^2 \tag{2}$$

or,

$$\rho_e(k) = [\nu(2m)^{3/2}/2\pi^2]\ [-V(k)]^{1/2} \cdot \left[-\frac{dV(k)}{dk}\right] \tag{3}$$

In a modified 'Thomas-Fermi' [6] model adapted to the case of a QGP droplet, the electrons get replaced by quarks which are also Fermions, and the minimum kinetic energy of the quarks at each point in phase space must exceed the confining/ de-confining potential at that point, since the QGP by definition is a deconfined gas of relativistic quarks and gluons as against the non-relativistic electron of the conventional Thomas-Fermi Model. Therefore, $p_{min} = [-V_{conf}(k)]$ and $p_{max} = [-V_{conf}(\infty)]$ which represents a reference energy and can be set to zero, remembering that we are dealing with a relativistic system and where 'k' refers to the corresponding quark momenta in phase-space. So an expression similar to (3) holds for the quark density of states, with the replacement of $V(k)$ with a suitable QCD induced phenomenological potential. The quark density of states therefore is

$$\rho_q(k) = (\nu/\pi^2)[-V_{conf}(k)]^2 \left[\frac{dV_{conf}(k)}{dk}\right] \tag{4}$$

The physical situation of the modified 'Thomas-Fermi' QGP droplet is illustrated by the phase diagram in Fig. 1. In this adaptation of the "Thomas-Fermi" idea, we only capture the spirit of the original idea for a system which is very different in detail. The primary difference between the electron-gas cloud surrounding the Thomas-Fermi nuclei and the Q.G.P. is the presence of the central potential in the formar and the many-body Q.C.D. potential in the latter, apart from the thermodynamically cold nature of the former system as against the hot plasma with the hydrodynamical flows in the latter. With all these differences in the background, we can still use the Thomas-Fermi density of state (3) as a template to construct the quark density of states in a Q.G.P. with suitable parametrisation to take care of the hydrodynamical (plasma) characteristics of the Q.G.P. as we introduce in the next section. Actually, the earliest and most successful application of a statistical model to a system other than cold electrons is the famous Bethe density of states for the nucleons [6] . Of course, our quark-density of states is a further extension in that direction.

3 The phenomenological inter-quark potential and the Free energy

The dynamical nature of the quarks and gluons in Q.G.P. forces us to seek an interquark potential which can account for the bulk properties (thermodynamical) of the quarks and gluons. The thermal mass formalism and the corresponding thermal Hamiltonian in the litrature [4,5] leads us to the following choice for the confining/de-confining potential

The "Thermal-Hamiltonian" for the Q.G.P. is [4,5]

$$H(k,T) = [k^2 + m^2(T)]^{1/2} \equiv k + m^2(T)/2k \text{ for large k or}$$
$$= k + m_0^2/2k - \{m_0^2 - m^2(T)\}/2k \tag{5}$$

where

$$m^2(T) = \gamma_{g,q} g^2(k) T^2 \tag{6}$$

with k the quark (gluon) momentum, m_0 the dynamic rest mass of the quark, T the temperature and $g(k)$ for first order. Q.C.D. running coupling constant, which for quarks with three flavors is,

$$g^2(k) = 4/3.12\pi/27.1/ln(1 + k^2/\Lambda^2) \tag{7}$$

with the Q.C.D. parameter $\Lambda = 150MeV$. $\gamma_{g,q}$ is the phenomenological parameter which we take as $\gamma_q = 1/6$ [4],while γ_g varies around the value of γ_q. The third term in (5) can be interpreted as an effective thermal potential for the Q.G.P. which has the form:

$$V_{\text{eff}}(k) = (1/2k)\gamma_{g,q}\ g^2(k)T^2 - m_0^2/2k \tag{8}$$

The main advantage of this parametrisation is that it fits nicely with lattice Q.C.D. simulations [4,5].

Since the Q.G.P. is a deconfined gas of quarks and gluons, the momentum of the patricles exceed the potential at each point in phase space, whereby

$$k_{min} = V(k_{min}) \qquad \text{or} \qquad k_{min} = (\gamma_{g,q} N T^2 \Lambda^2/2)^{1/4} \tag{9}$$

where $N = (4/3 \times 12\pi/27)^3$

The existance of k_{min} leads to a natural low energy cut off in the model leading to finite integrals by avoiding the infra-red divergence. It is interesting to note taht the k_{min} is of the same order of magnitude as Λ and T. This is unlike in the models of earlier authors who introduce the cut off in a rather ad-hoc fashion [1,2].

Thus for evaluation of density of states of the quarks we have to evaluate relation (4) after introducing potential (8) in it. The gluons are also confined in the hadrons and deconfined in the QGP, we impose the low energy cut off for the gluon density of states as well as this takes care of the consistency of the treatment of both the gluon and quark sectors.

For the free energy we use the usual continuum expression for a system of moninteracting fermions (upper sign) or bosons (lower sign) at temperature T, we have

$$F_i = \mp T g_i \int dk \rho_i(k) \ln(1 \pm e^{-(\sqrt{m_i^2+k^2})/T}) \tag{10}$$

where $\rho_i(k)$ is the density of states of the particular particle i (quarks, gluons, interface, pions etc.) being the number of states with momentum between k and $k+dk$ in a spherically symmetric situation, and g_i is the degeneracy factor (color and spin degeneracy) which is 6 for quarks and 8 for gluons and one for pions and the interface.

Unlike the assumption of the earlier authors [1,2], the interfacial surface is no longer a MIT bag, and yet it has a contribution to free energy on account of the surface energy which we assume to be a scalar Weyl-surface [5] in our approach with suitable modification to take care of the hydrodynamic effects at the surface. Therefore, the interface free energy is

$$F_{interface} = \gamma T \int dk \rho_{weyl}(k) \delta(k - T) \tag{11}$$

Where γ is the parameter which takes care of the hydrodynamical effects and is chosen to be

$$\gamma = \sqrt{2} \times \sqrt{(1/\gamma_g)^2 + (1/\gamma_q)^2}, \tag{12}$$

which is the inverse r.m.s value of the phenomenological flow- parameters of the model.In fact we could have chosen γ as an independent parameter,but in

order to limit the number of free parameters we make a conscious choice of this formula.
The Weyl density of state is

$$\rho_{weyl}(k) = (4\pi R^2/16\pi)k^2 \tag{13}$$

'R' being the radius of the droplet.
Therefore,

$$F_{interface} = \frac{1}{4}\ R^2\ T^3\gamma \tag{14}$$

The colour degeneracy g_i is 6 for quarks and 8 for gluons. We evaluate the free energy at the temperature $120\ MeV < T < 250\ MeV$ at $10 - 20\ MeV$ to have a ready comparison with the earlier papers.

The pion free energy is [2]

$$F_\pi = (3\ T/2\pi^2)\nu \int_0^\infty k^2 dk \ln(1 - e^{-\sqrt{m_\pi^2+k^2}/T}) \tag{15}$$

For the quark masses we use the curent (dynamic) quark masses $m_0 = m_d = 0\ MeV$ and $m_s = 150\ MeV$, just as in reference [2].

Results and Conclusion :

With all the above numerical and theoretical inputs we have computed the free energy contributions of the u + d quarks, s-quarks and for the gluons while retaining the same behaviour for the pions as in [1,2]. All the energy integrations involved for the quark sector have a low energy cut-off at 129 MeV by virtue of (9), and the integral saturates at an upper cut-off at nearly four times the low energy cut-off.

In the present approach the bag energy is replaced by the interface energy (14) and the individual Free-energy contributions are shown in $FIG.2$ for a particular temperature vig. $T = 152\ MeV$ for $\gamma_g = 12\gamma_q$. The behaviour of the total Free-Energy of the droplets with increasing droplet size for various temperatures in the range $120\ MeV < T < 250\ MeV$ for the various sets of flow-parameters $\gamma_q \leq \gamma_g \leq 12\gamma_q$ with $\gamma_q = 1/6$ (Peshier -Value) are illustrated by $FIG.3$ to $FIG.8$.

It can be seen that the QGP-droplet-Hadron Free-energy goes on increasing without any stable droplet forming for a choice of the flow-parameters $\gamma_q \leq \gamma_g \leq 6\gamma_q$,with γ_q fixed at the value 1/6 as is evident from the graphs $FIG.3\ to\ FIG.5$.

Large stable Q.G.P. droplets of $R > 6\ fm$ start appearing for the value of $\gamma_g = 8\gamma_q$ at $T > 230\ MeV$ [FIG.6]. Stable Q.G.P.droplets with smaller radii less than $6\ fm$ start appearing for a choice of $\gamma_g > 10\gamma_q$, albeit with much lower barrier heights indicating that the larger droplets are highly unstable and the QGP-hadron phase transition occurs at lower temperatures of $T \sim 170\ MeV$ [FIG.7 to FIG.9]. At $\gamma_g > 16\gamma_q$ [FIG. 9] the droplets become highly unstable with the barriers height almost vanishing, so that the system spontaneously passes into a QGP phase without the intermediate state of QGP droplet formation at much lower temperatures of $T < 100\ MeV$. The crucial role played by the hydrodynamical flow-parameters indicates both their need and primacy in adapting a statistical model meant for a cold system of electrons or nucleons to an essentially hot plasma system of QGP. Also the smooth cut at the phase boundary is indicative of a first -order phase transition as suggested by earlier authors using other models [1,2]. In short the model gives a simple and robust mechanism for the transition from the hadronic phase to the Q.G.P. phase with a minimal phenomenological input in terms of the hydrodynamical flow-parameters and the current quark masses. But as to which of the scenario occurs in actuality , only experiments can tell.

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Figure Captions :

FIG. 1 : The Phase space picture of the "Thomas-Fermi" oriented model of QGP droplet in a bulk pionic medium .

FIG. 2 : Individual contribution to Free - energy from the quarks, gluons, pions and the interface leading to the total Free - energy at $T = 152\ MeV$.

FIG. 3 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 2\gamma_q$ and $\gamma_q = 1/6$.

FIG. 4 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at differnt temperatures for the flow-parameters $\gamma_g = 4\gamma_q$ and $\gamma_q = 1/6$.

FIG. 5 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 6\gamma_q$ and $\gamma_q = 1/6$.

FIG. 6 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 8\gamma_q$ and $\gamma_q = 1/6$.

FIG. 7 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 10\gamma_q$ and $\gamma_q = 1/6$.

FIG. 8 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 12\gamma_q$ and $\gamma_q = 1/6$.

FIG.9 : The variation of total Free-energy F_t of the QGP droplets in a pionic medium at different temperatures for the flow-parameters $\gamma_g = 16\gamma_q$ and $\gamma_q = 1/6$.

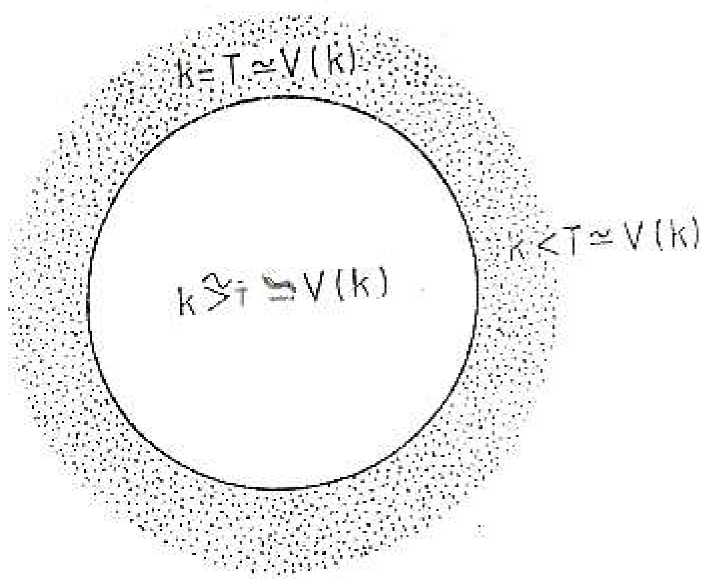

FIG. 1

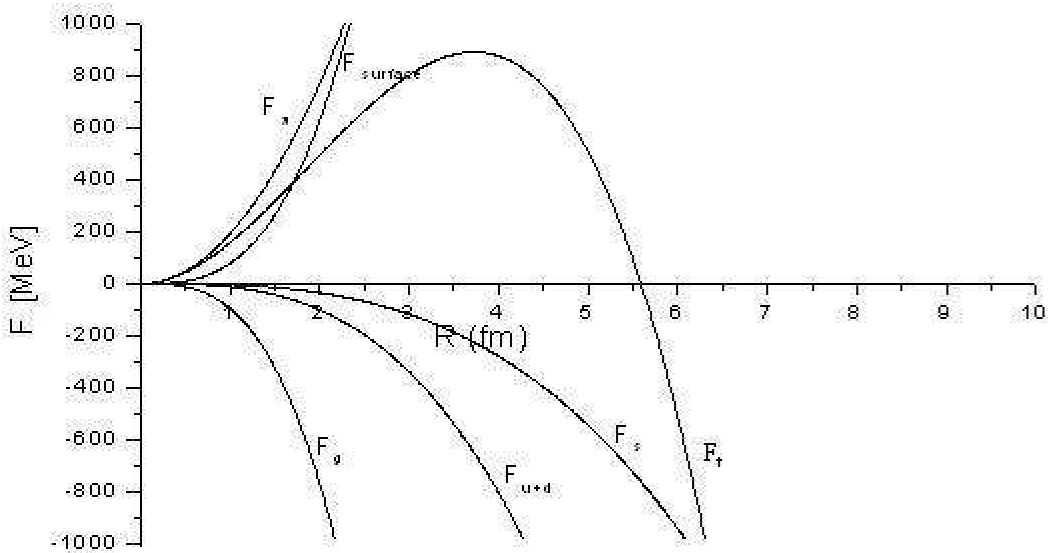

FIG. 2: At $\gamma_g = 12\gamma_q$, $\gamma_q = 1/6$ for Temperature T= 152 MeV

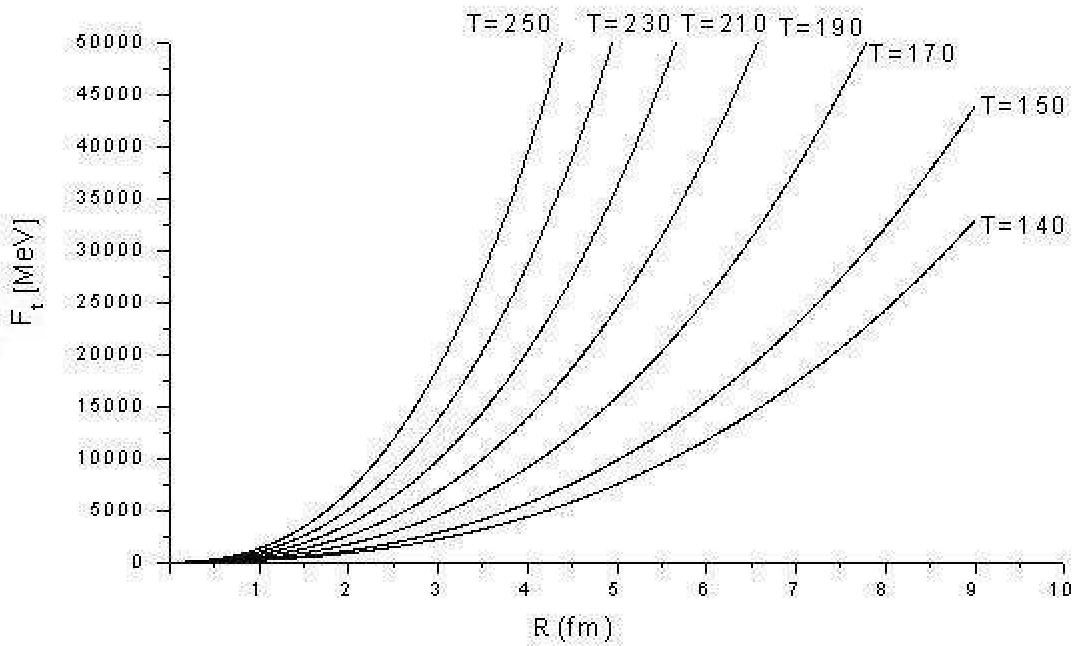

FIG. 3: At $\gamma_g = 2\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.

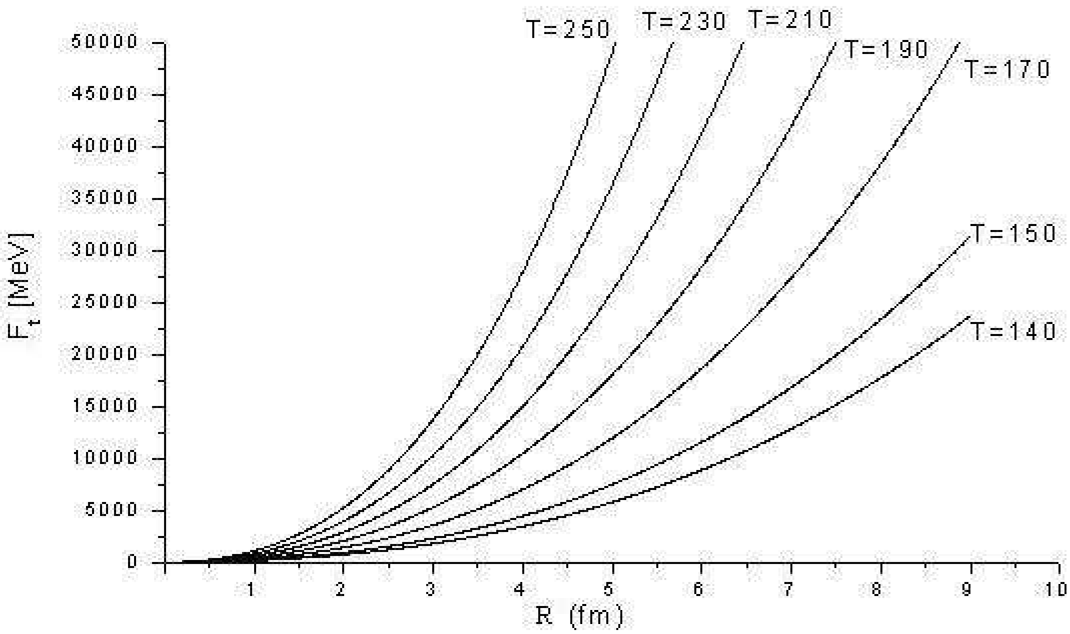

FIG. 4 : At $\gamma_g = 4\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.

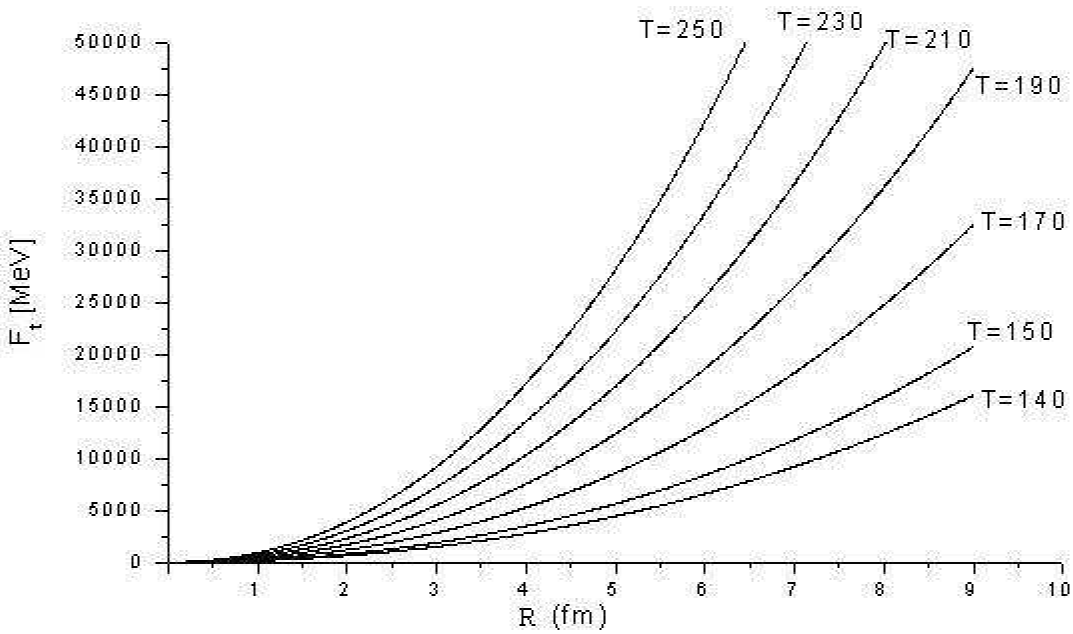

FIG. 5 : At $\gamma_g = 6\gamma_q$, $\gamma_q=1/6$ for various Temperatures.

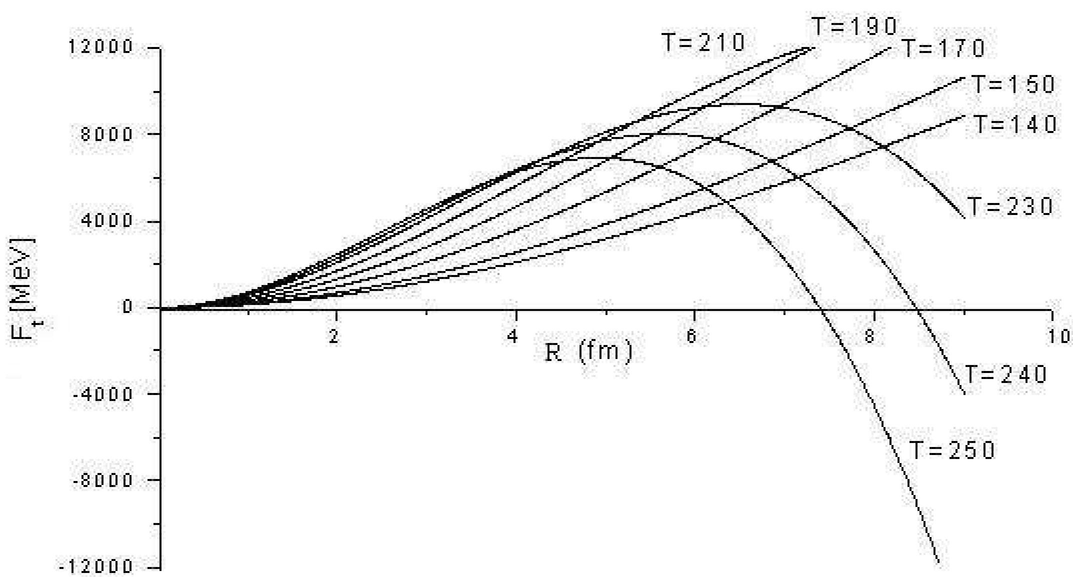

FIG. 6 : At $\gamma_g = 8\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.

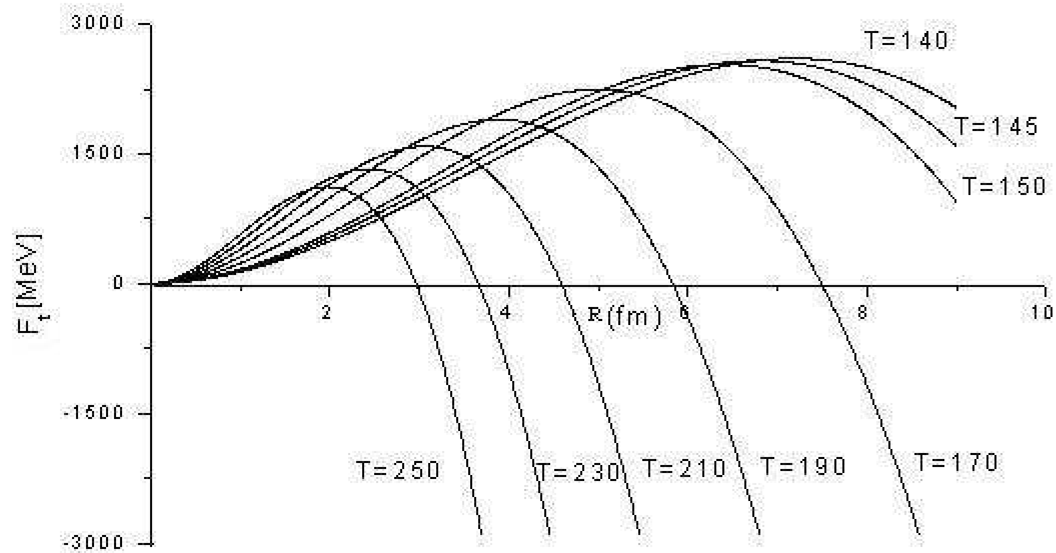

FIG.7 : At $\gamma_g = 10\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.

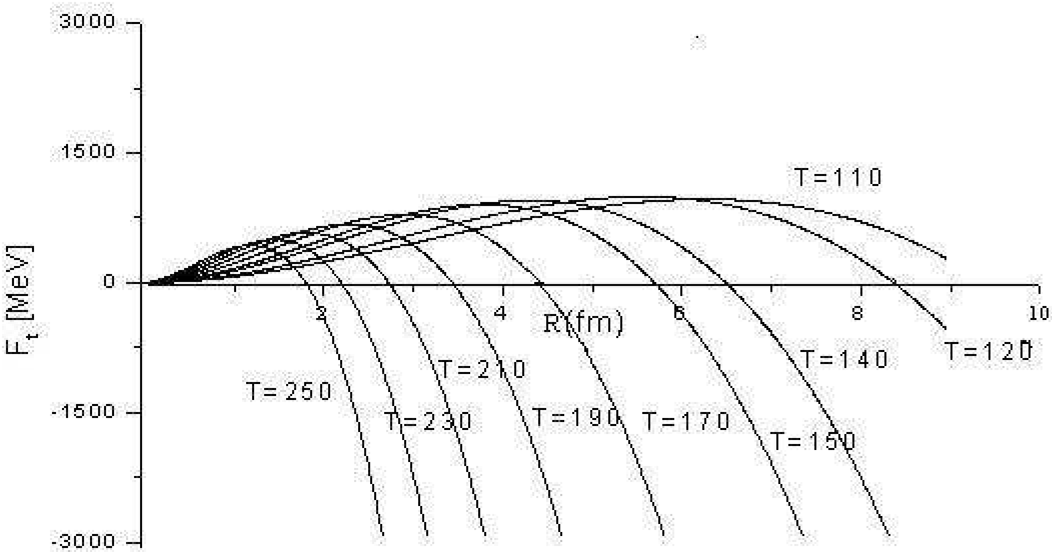

FIG. 8: At $\gamma_g = 12\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.

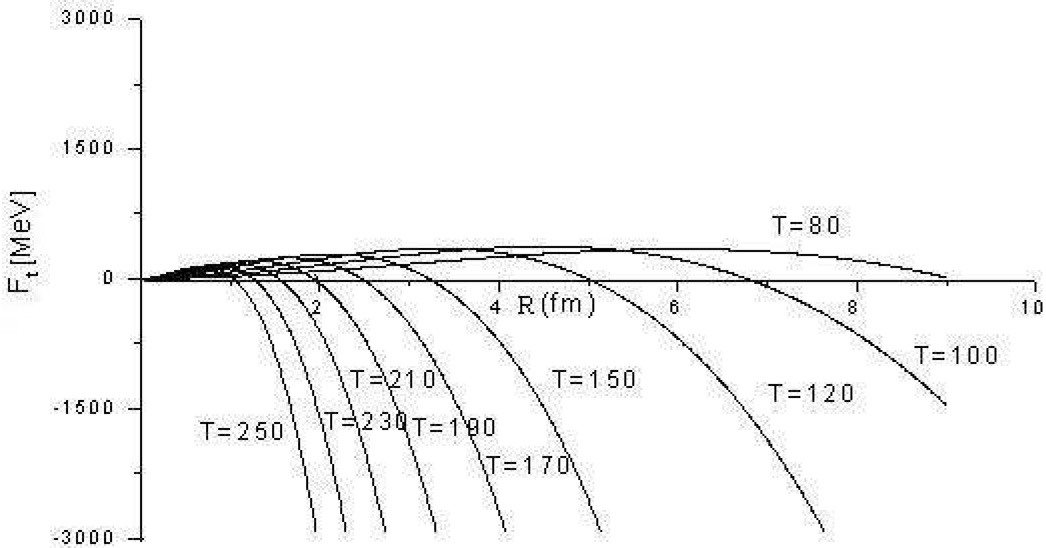

FIG. 9: At $\gamma_g = 16\gamma_q$, $\gamma_q = 1/6$ for various Temperatures.